Understanding the Unit Circle:

Increasing the

Level of Student Responsibility

Through Use of Exploration Activities

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Problem Statement

The ultimate goal for any teacher is for students to learn the subject matter that they are responsible for. However, there are different definitions and levels of "learning." Can we say that a student has successfully learned the material if they can apply the appropriate algorithm and come up with the correct numerical answer, yet they do not understand why or how? Have we achieved learning if a student can regurgitate facts and definitions if they do not know what the definitions mean or how these facts and definitions can be used to solve real-world problems? These are questions that teachers are faced with in their classrooms on a daily basis.

According to the National Council of Teachers of Mathematics Standards, "The vision of school mathematics in *Principles and Standards* is based on students' learning mathematics with understanding." (NCTM, 2000). According to the NCTM, it is not sufficient for students to have factual knowledge and procedural proficiency without conceptual knowledge. It is the combination of facts, procedures and concepts that produce true mathematical understanding.

When students only have factual or procedural knowledge, they lack the flexibility needed to solve complex problems and are unable to apply what they know across a variety of different types of problems. Solving complex problems requires the use of all prior knowledge. When students develop factual and procedural knowledge without understanding, they do not know how all of the things they have learned in the past relate to each other. The result is the inability to use their prior knowledge to solve more than one type of problem. I have seen this situation in my own classroom many times. Instead of trying to understand the concept, students try to find an equation or algorithm that they can use every time to solve that type of problem. They often appear to be successful because they can come up with the correct answer when the problems are presented in the same manner as those used to teach the algorithm. When the problem is presented in a slightly different manner or problems of another type are also presented, students do not know how to proceed. They are lacking the conceptual knowledge that supports the use of the procedural knowledge.

One of the biggest challenges that I have faced in teaching students mathematics to achieve understanding is the students' willingness to take responsibility for their own learning. Often students are looking for an algorithm or procedure that they can memorize to get by and are content to let someone else do the thinking for them. They feel that as long as they can produce the correct answer on the homework that they have "learned" the material and do not need to work any harder to develop a deeper understanding of the content. For students the ultimate goal is to complete the assignment where, as a teacher, my goal is for them to develop a deeper understanding of the concept that they can apply across a variety of problem types and situations.

Nowhere has this lack of conceptual knowledge been more evident for me than in teaching students in Algebra II about the unit circle. I have taught this concept many times over the past several years and have had limited success with students developing a true understanding of the concept of the unit circle and how it relates to trigonometry.

Literature Review

The literature review that I conducted for this project definitely shaped the development of the problem statement and more specifically the method for achieving the end goal. The many articles and other resources that I reviewed definitely supported the importance of student responsibility in developing the conceptual knowledge required to be a flexible problem solver. Originally, I was going to use error analysis as a means for increasing student responsibility and improving conceptual understanding, however, much of the literature I read suggested that the use of student-centered explorations was also an effective way to encourage students to take the responsibility that is necessary to acquire this conceptual knowledge.

As I conducted the literature review for this project, I thought back on various lessons and topics that I had taught over the years. One specific topic that comes to mind is the unit circle in Algebra II. I have taught this topic several times and have never been happy with my students' conceptual understanding. Many times they have attempted to memorize the angle measurements and corresponding ordered pairs only to be unsuccessful at being able to use the information to solve problems. This has lead me to research using student-centered explorations to develop conceptual understanding versus my traditional direct instruction method which, in the past, has only developed procedural knowledge and fact memorization.

The first resource that I explored was the NCTM Principles and Standards for School Mathematics. I was already fairly familiar with this particular resource and knew that I would find support for the use of student-centered activities to develop true mathematical understanding. This resource provided me with a very broad picture of what conceptual understanding looks like and what type of activities could help develop it.

A second resource that was very helpful in designing this project was "Problem Posing: An Opportunity for Increasing Student Responsibility" by Robert Cunningham. Although this article focused primarily on using the strategy of having students pose their own problems to support learning about a concept, it did provide me with significant insight into what makes students take an active part in their own education. Cunningham states that "The Instrumentalist perspective emphasizes the mastery of skills, rules, and procedures that become ends in themselves, rather than devices for understanding, representation, and reasoning. From this perspective, the student's role is to master what the teacher says. When mathematics instruction mimics these narrative perspectives, the textbook or teacher is continually relied upon for problem generation and for explicit instructions on how to solve problems. Students serve mostly as listeners and have little responsibility for constructing their own knowledge." (Cunningham, 2004, p. 83). He goes on further to say that "…students need to develop a problem-solving disposition, one in which they realize that it is common to experience difficulty when presented with an unfamiliar situation. If students are to eventually explore such situations on their own, and utilize past learning, then opportunities to do so must be provided." (Cunningham, 2004, p. 89). In essence, Cunningham feels that for true learning to occur, students have to reduce their dependency on the text-book and give up the notion that the teacher is the ultimate authority. They need to encounter mathematics that challenge them and push them to think for themselves. I knew that I needed to develop an activity where students could not find the answers they needed in their textbook and one in which they had to find the answers through their own exploration.

Another resource that was extremely helpful was "What's the Buzz? Tell Me What's Happening! Developing Ownership of Understanding in Mathematics Education" by Joseph Spadano. Spadano's article describes two opposing views on student instruction. On the one hand, you have the teacher-centered, social behaviorists who view teaching as "transferring generalized knowledge through pre-determined, outcome-based objectives." (Spadano, 1996, p. 2). They are satisfied that learning has occurred when "mastery of objectives through skillful use of algorithms has been accomplished and measured by criterion-referenced exams." (Spadano, 1996, p. 2). The student of the social behaviorist has faith in what the teacher knows, but has not experienced if for him or herself. On the other hand is the student-centered, experientialist. The experientialist believes that learning has occurred when students have taken ownership for developing their own understanding. This understanding comes from the experience of doing. (Spadano, 1996). The social behaviorist gives students the algorithm while the experientialist helps students develop the algorithm on their own.

Spadano also talked about developing a sense of responsibility in students. He feels that in order for students to take responsibility for their learning, they have to be given the opportunity to become autonomous, independent learners. In a teacher-centered environment, students take "on faith" what the teacher knows. They may be held accountable for doing the homework or providing the correct answer, but they are not responsible for developing understanding. That is done for them by the teacher.

For students to really become responsible for their own understanding, they must be given the freedom to explore for themselves. Spadano's article confirmed for me the need to provide students with an activity for the unit circle that would help them to develop their own understanding.

In most cases, I have taught from a teacher-centered perspective. I have struggled with the balancing act of teaching for understanding by using student-centered methods and covering all of the material that is required through use of direct instruction. My experience has been that student-centered activities take much more time to complete than direct instruction, and every year the amount of content that I have to cover in class has increased. Because of this, most of the students in my class are not familiar with doing the work of a student-centered exploration. I needed an additional resource that would help me figure out the best way to transfer responsibility for learning to my students.

"Releasing Responsibility" by Douglas Fisher and Nancy Frey gave me a lot of information about how to make the transfer of responsibility. Fisher and Frey stated that "we must give students supports that they can hold on to as they take the lead – not just push them onto the path and hope they find their way." (Fisher & Frey, 2008, pg. 33). The first thing they suggested for giving students support was making sure that the learning objectives and purpose of the activity are clearly communicated to the students. Students are more likely to fully participate in an activity when they understand why they are doing it. The next recommendation was for the teacher to model the type of thinking and language that the task will require. The third recommendation was that new student-centered tasks be conducted in a group setting.

In a small group situation, students are more likely to take risks than they are in a whole class environment. The last suggestion was to use guided instruction for individuals who need additional support in engaging with the task.

One final resource that provided me with insight into how to conduct my research was Magdalene Lampert's book "Teaching Problems and the Problems of Teaching." I found Chapter 10 "Teaching Students to Be People Who Study in School" to be particularly helpful. One thing that influences how a student takes responsibility for their own learning is how they seem themselves as a learner. Very often, students do not see themselves as being "good at math" and this impacts how hard they are willing to try to truly understand. Lampert states, "If a student does not seem himself or herself as the kind of person who is going to learn, it seems unlikely that learning will occur. Even if such a person were tricked into acquiring some knowledge or skill, the chances that such learning would ever be used in public are probably slim." (Lampert, 2001, p. 266). This was one aspect of student responsibility that I had not previously considered. Prior to this project, it did not dawn on me that a student may not want to take responsibility for developing conceptual learning because they did not want to be responsible if they failed.

Modes of Inquiry

I decided to approach the problem by developing an activity for the students to complete in pairs that would allow them to explore the unit circle on their own. On the one hand, I wanted the activity to be something that would allow them to make conjectures and explore whatever avenues their curiosity took them down. On the other hand, by the end of the activity, I needed them to make specific connections between the unit circle and the three main trigonometric functions we had been studying. I tried to develop the activity in such a way that many conjectures were possible yet making some specific comparisons would lead everyone to the same understanding of the unit circle. I also tried to make an activity that would require students to use a significant amount of prior knowledge to answer the questions.

As previously mentioned, I had considered using error analysis to encourage students to take responsibility for developing a conceptual understanding. I was originally going to do the error analysis research (and probably still will) with my Algebra I class while we are studying parabolas. While I feel there is value in doing error analysis, developing conceptual understanding of the unit circle is something that has bothered me for years, and I saw this as an opportunity to address this concern. I may also use error analysis to eliminate any misconceptions that students still have about the unit circle later in the course.

Prior to the activity, I wanted to make sure that students understood the objective and purpose of the activity. I explained to them that I have taught this material several times in the past and have not been happy with the connections that students have made. I posted the objective at the top of the activity sheet so students had it in front of them as they worked. Because students in my classes are not extremely familiar with student-centered activities (and because I anticipated that they were going to have a difficult time getting started), I continued the activity by modeling the type of thinking that was going to be required. I showed them how to approach the first question in the activity and what types of explanations I was looking for. Students in math classes are not used to doing much descriptive writing, so I wanted them to understand that their answers to the questions needed to be written in complete sentences and have a significant amount of detail.

The activity started out with having the students solve the unit circle equation for y and put the results into their calculator. Students were then asked to write down the coordinates of two ordered pairs per quadrant that were on the circle. Along the way they were asked to make observations about the equations they developed when they solved the unit circle equation for y and about the table from which they identified their ordered pairs. Each of these areas provided several opportunities for making observations and connections to other previously studied topics like domain and range and the imaginary numbers.

In the next part of the activity, students were required to plot the ordered pairs on the coordinate plane and then draw a ray from the origin through each ordered pair. Using a protractor, they measured and recorded the angle that each ray made with the positive x-axis. This part of the activity not only refreshed their skills in using a protractor, but also gave them additional experience with angles of rotation which was a topic that we had been studying the prior day.

Once each angle had been measured, students were asked to fine the sine and cosine of the angle using their calculators. We had spent two days prior to the activity working on finding the values of the three main trigonometric functions for a given angle. I was hoping that this would tie together the trig functions with the angles of rotations.

Students were then asked to make a comparison between the original table of ordered pairs and the trig function values. They had to make a conjecture as to how they thought the two tables related to each other. At this point students should have developed the conceptual understanding that the x-values of the ordered pairs on the circle were the same as the cosine values for the angle of rotation, and that the y-values of the ordered pairs on the circle were the same as the sine values for the angle of rotation. In the past, I have shown students this relationship and have tried to make the connection for them with little success.

It was not enough for students to understand that there was a connection between the ordered pairs on the circle and the sine and cosine values for the angle. I wanted them to understand why these values were connected. The next step in the activity was for students to take one point in each quadrant and draw a right triangle, using the x-coordinate of the ordered pair for one leg and the y-coordinate of the ordered pair for the other leg. Once they had the triangle drawn, they needed to find the length of the hypotenuse using the Pythagorean Theorem. Students were then asked to calculate the sine, cosine and tangent of the angle of rotation using the definitions for each function. Because these points were all on a circle of radius 1, the hypotenuse for each triangle was 1.

For the last part of the activity, students had to determine the relationship between sine, cosine and tangent and use the information they found in the previous step to prove or disprove their conjecture about how the ordered pair values and the sine and cosine values were related.

Results

The activity for this project was conducted during two of my Algebra IIB classes during this trimester. Approximately 60 students completed the activity which lasted over two days. We started the activity on a Friday and completed the wrap-up and follow-up assignment during the class session on Monday.

Data collection for this project was somewhat difficult. During the activity I made observations about how the students were doing conceptually with the material as well as how they were doing with this particular type of activity. While I don't have any specific data to compare their conceptual understanding to, I do have a general notion of the conceptual understanding that students have developed in the past based on a follow-up assignment that I have used after teaching the unit circle. I also have experience with students at this level working on group activities.

Much of the analysis of the data occurred while the students were working on the activity. As I watched students work and heard the discussions that they were having with their partner, I compared it to the types of interactions I have had with students in the past regarding the unit circle. Having taught this concept several times before, I am aware of the specific misunderstandings and misconceptions that students have had in the past. During the activity I made several observations comparing my past experiences with the topic and the experience that my students were having with the activity. In the past, many of those misconceptions have come from the fact that the students have not had to work at developing their own understanding of the unit circle.

One thing that I observed during the activity is that students are definitely not accustomed to doing activities that do not lead them through the process. This activity was somewhat open-ended and allowed students to approach the material from a variety of different viewpoints and using a variety of different methods. When faced with this type of activity, the first reaction of many students was to throw their hands up and say, "I don't get it." During this activity, I was very deliberate in my reaction to this type of behavior. In the past, I have found myself somewhat encouraging their helpless behavior by giving in and giving them the answer. This time my response was, "Well, what do you think we should do?" By using this question, I was trying to encourage them to think for themselves and to recognize that there are other resources available to help them think through their solution. For many students, my response was frustrating; however, once they recognized that they would have to think through the activity for themselves, most of them focused on working in collaboration with their partner and other pairs in the class. This activity took two days to complete and there was definitely a decrease in the level of helpless behavior during the second class period.

Another thing I observed during the activity was that students were making many more connections between the parts of the activity and therefore, between the different concepts than they had in the past. My usual method of teaching the unit circle in the past was to present the circle to the students, showing them the ordered pairs for the 30°, 45°, 60° and 90° angles. I would then tell them that the cosine of the angle was the x-coordinate of the ordered pair and the sine of the angle was the y-coordinate of the ordered pair. Even though I would show them why it was true using right triangle trigonometry, many students did not make the connection and were then not able to really use the information to solve problems.

During the activity students investigated the connection between the coordinates and the trig functions for themselves and had to make a conjecture about what the connection was. Because they were able to see the connection for themselves, it made much more sense to them than it did when I would show them. They also had to try to prove or disprove their conjecture, and many of them recognized that they could show that it was true using the right triangle trig that they had learned during the previous section. After the activity, students were given several problems where they would have to apply the knowledge they gained during the activity to solve the problems. Because they had made the connections themselves, most students were able to easily complete the problems and explain their reasoning in developing the solutions. I have given the same problem assignment in previous years and most students struggled to solve the problems and were unable to provide much in the way of explanation as to why they gave the answer they did.

Overall, I feel the activity was fairly effective in getting students to take responsibility for their own learning, which in turn, improved their conceptual understanding of the unit circle. Most students actively engaged in the activity, and many more students were able to correctly apply the concepts of the unit circle during follow-up activities than have been able to in the past.

Conclusions and Limitations

From this inquiry, I learned that students are more likely to take responsibility for their own learning if: 1) they are encouraged to do so by the teaching and questioning methods that I use in the classroom, and 2) they are allowed to construct the knowledge of some concepts themselves instead of having it presented to them. The result is a much deeper understanding of the concept and an increased ability to apply the knowledge in a variety of different contexts.

While my observations of the effectiveness of this type of activity were directly supported by the literature that I reviewed, there are some limitations as to the usefulness of this research and this type of activity. Because of the nature of the activity, most of the data that I collected was anecdotal and subjective at best. It is very difficult to collect objective, statistically significant data in this type of activity. It is this type of data that is necessary to conclusively prove the effectiveness of the activity.

Another limitation of this type of activity is time constraints. I needed two days to complete the activity and an additional day to connect the learning from the activity to the exact value angles on the unit circle. Because this is an important topic for students who will be taking pre-calculus next year, this amount of time is warranted. However, not all topics are this important and, due to the large amount of material that must be covered in a short amount of time, not all topics can be taught using a student-centered exploration.

Next Steps

I learned a great deal during this inquiry that will change the way I teach. Because of the effectiveness of this type of activity during this project, I would like to incorporate more student-centered explorations into all of my classes, including some of my lower level Algebra I classes. I feel this will be a very effective method of helping struggling students develop the key concepts that they need to be successful as they continue their mathematics education. I will need to focus my efforts on the concepts that are the most important and will benefit students the most.

I also learned that how I respond to students influences how much responsibility they are willing to take for their own understanding. If I consistently tell students the answers without pushing them to think for themselves, they develop a sense of learned helplessness. By asking them tough questions and encouraging them to use additional resources, I can help them develop habits of responsibility that will help them in life, not just in math class.

Now that I have seen how important student responsibility is in terms of conceptual knowledge development, I would like to pursue additional methods of encouraging students to take the responsibility. One method that I am interested in is through analysis of errors. By using the types of errors that students typically make, I would like to teach students to analyze their own work and to identify conceptual misunderstandings that caused the error. Once identified, we can then correct the misunderstanding.

References

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